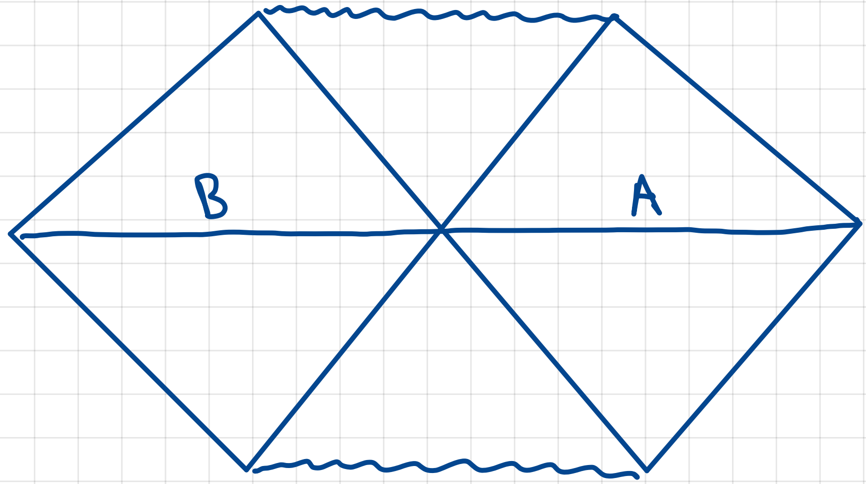
7.

Hawking Radiation

Read: QGBH §5

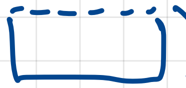
"Eternal" Schwarzschild BH

(i.e. maximal analytic extension - not formed by collapse)



cf. Minkowski
→ Rindler !

- Killing vector ∂_t, Killing Horizon, Bifurcation pt.

We'll <u>define</u> the "Hartle-Hawking" state by a Euclidean P.I. (cf. Minkowski vacuum $|0\rangle =$ )

We'll see it's thermal, like Unruh.

Caveat: this is a choice of state. It is not yet a derivation of any physical effect.

Temperature

(First a quick trick, then a more careful path integral derivation.)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2$$

Change coordinates to go near $\mathcal{H}$:

Defn. R, η by

$$r = 2M\left(1 + \frac{R^2}{16M^2}\right)$$

$$t = 4M\eta$$

Expand $R \ll M$, ie $r \sim 2M$ near horizon $\Rightarrow$

$$ds^2 \approx \underbrace{-R^2 d\eta^2 + dR^2}_{\text{Rindler!}} + 4M^2 d\Omega_2^2$$

$$\eta \sim \eta + 2\pi i$$

$\Rightarrow$

$$t \sim t + 8\pi M i$$

$\Rightarrow$

$$\boxed{T = \frac{1}{8\pi M}}$$

(w.r.t. ordinary $E = Q[\partial_t]$)

Check 1^{st} law:

$$S = \frac{1}{4} \text{Area}$$

$$= \frac{1}{4} 4\pi (4M^2) = 4\pi M^2$$

$$dS = 8\pi M \, dM$$

$$T \, dS = dM$$

✓

To see this really T, define HH state by Euclidean P.I.

Euclidean BH:

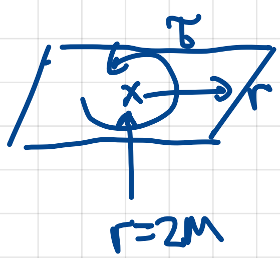
$$t \to i\tau$$

$$ds^2 = \underbrace{\left(1-\frac{2M}{r}\right)d\tau^2 + \frac{dr^2}{1-\frac{2M}{r}}}_{\text{Draw this}} + r^2 d\Omega^2$$

$$\tau \sim \tau + 8\pi M$$

Near horizon:

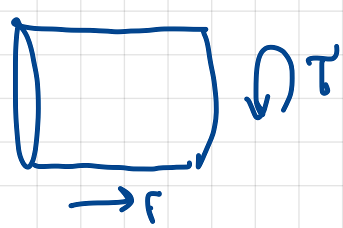
$r \sim 2M$:



Far away:

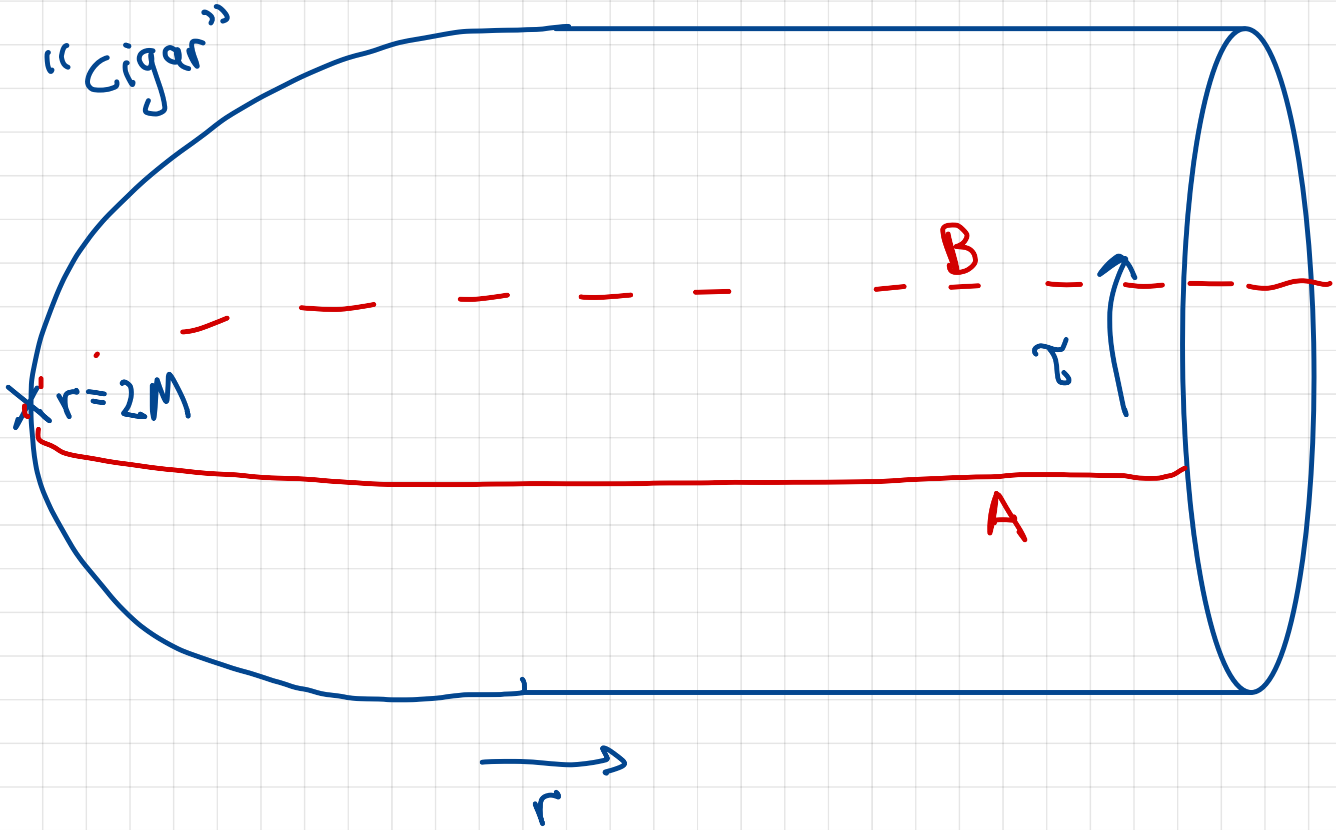
$r \gg 2M$

$$d\tau^2 + dr^2 =$$

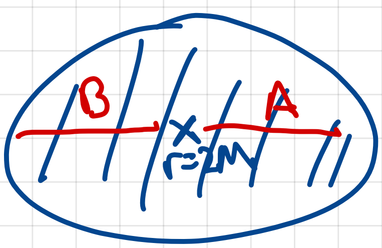


So:

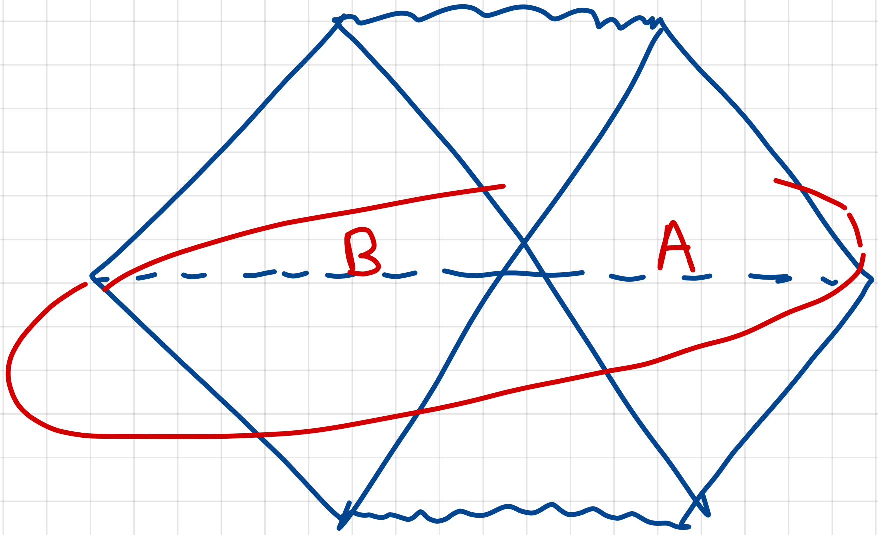
"cigar"



topologically,

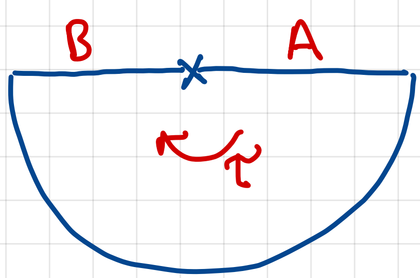


How does it match onto Lorentzian spacetime?



How do we know that B in cigar is B in Lorentzian? Because near the bifurcation point, it's Rindler vs. Minkowski

Path integral on the cigar prepares thermal state:

$|HH\rangle$ = 

"Hartle-Hawking"

$= |\beta = 8\pi M\rangle$

$$\rho_A^{HH} = \mathrm{tr}_B |HH\rangle\langle HH|$$

$$\boxed{\rho_A^{HH} = e^{-8\pi M H}}$$

* Unlike Unruh effect, this H is <u>ordinary</u> energy $= Q[\partial_t]$
  as measured by inertial observer far away.

* HH state defined by Euclidean P.I. is
  thermal in region outside.

* BH in thermal equilibrium w/ a hot gas
  (Euclidean PI $\leftrightarrow$ equilibrium!)

* Caveat: Thermal ensemble in ∞ Volume
  diverges. Need a "box" to regulate IR divergence.

* According to infalling observer, $|HH\rangle$ is empty @ horizon (like $|0\rangle_M$)

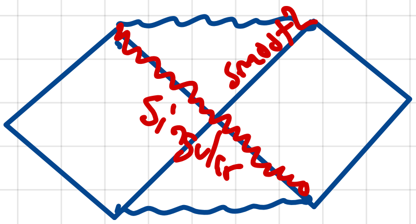
crucial for discussion of collapsing BH in a few minutes!

* Alternatively, $|0\rangle_{\text{Boulware}}$ $\equiv$ empty according to fixed-r observer "hovering" (like $|0\rangle_{\text{Rindler}}$)

But singular on $\mathcal{H}$

(because no entanglement across $\mathcal{H}$)

* $|\text{Unruh}\rangle$ $\equiv$ Thermal for outgoing, Boulware for ingoing

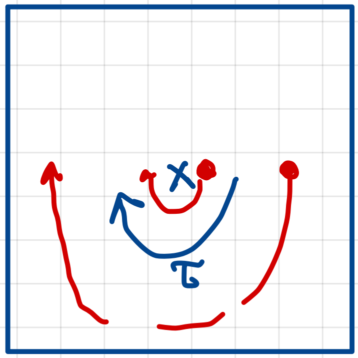


(non-equilib.)

How to think about the HH state:

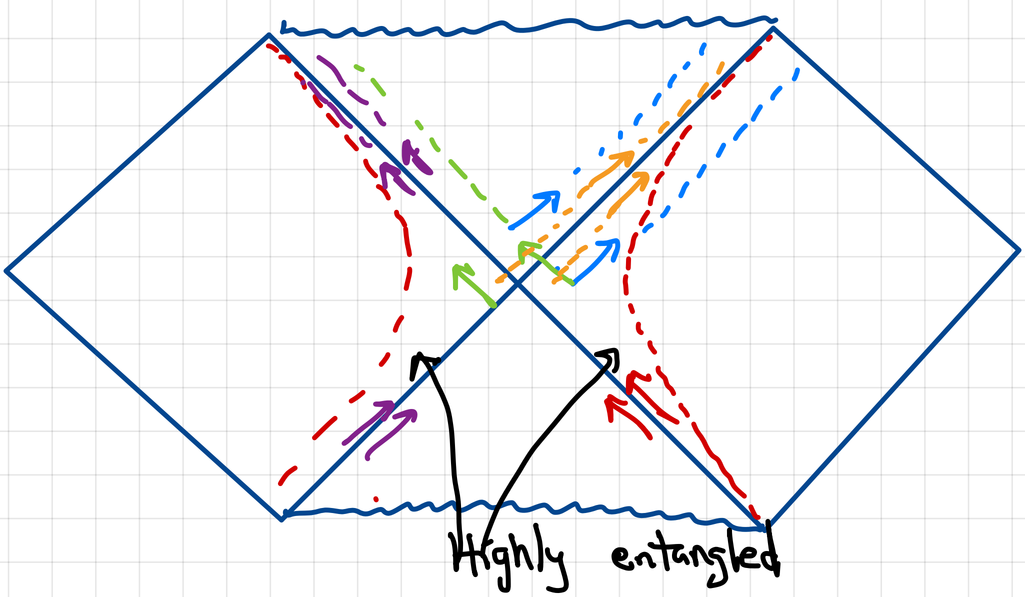
(Same applies to Minkowski vs. Rindler)

In Euclidean, stuff near origin has small $Q[\partial_\tau]$:



So $\sum e^{-\pi\omega} |n\rangle|n\rangle \sim \sum |n\rangle|n\rangle$ near origin.

In Lorentzian:



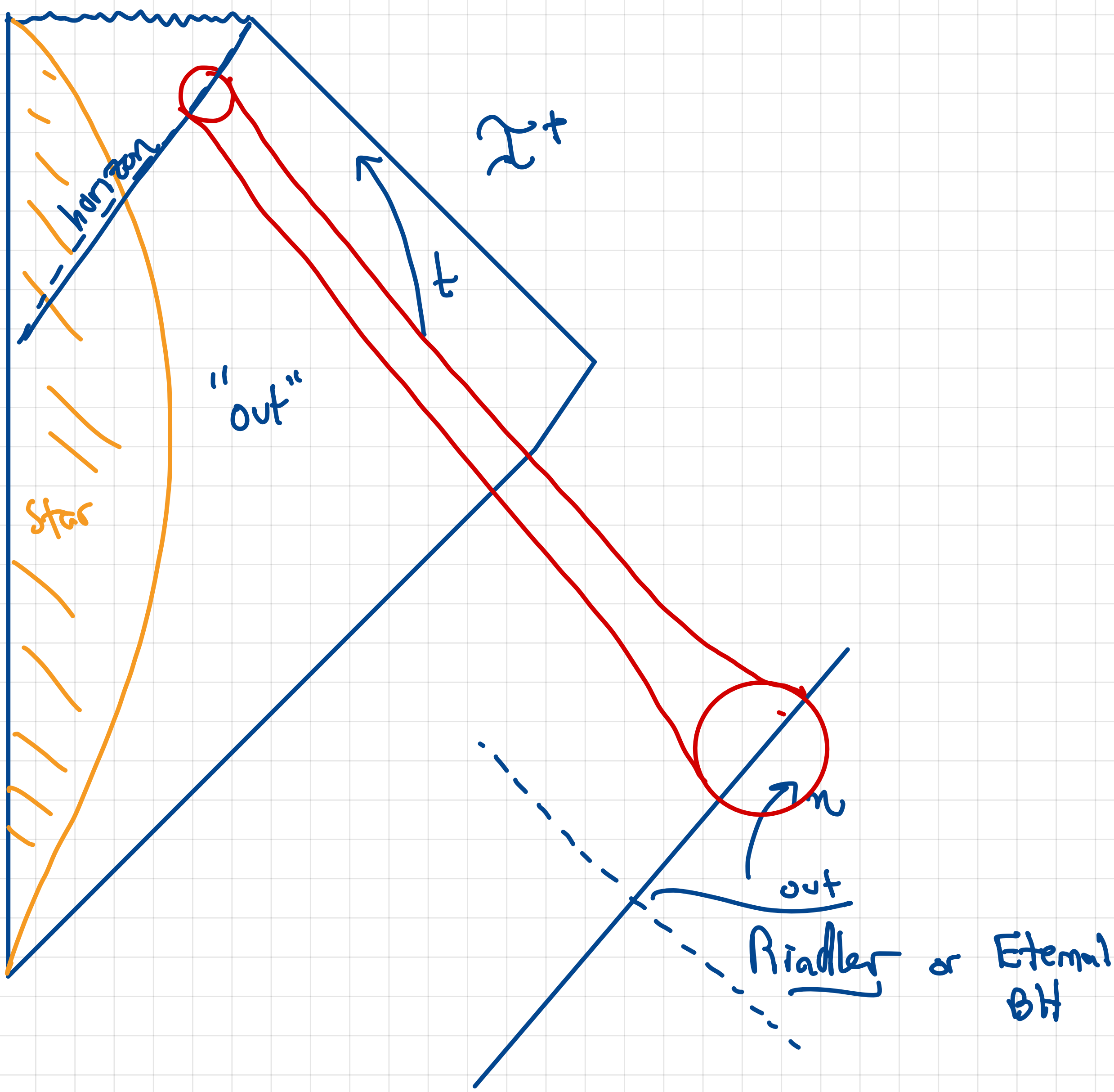
(This is easier to explain directly in Lorentzian: near horizon stuff is redshifted, so populated in $e^{-\beta H}$)

$|HH\rangle$ consists of highly entangled modes pointing along and "straddling" the 4 segments of $\mathcal{H}$.

Freefaller sees this as "vacuum". However @ ∞ see it as incoming radiation in equilibrium w/ outgoing radiation.

Black Holes formed by collapse

Classical Penrose :



- Metric outside = Schwarzschild (Birkhoff)
- No "left" region.

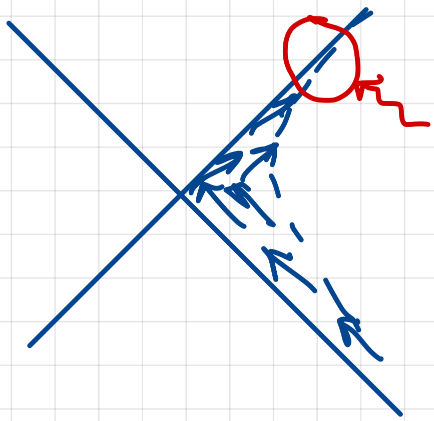
Equivalence Principle $\Rightarrow$ Freefaller sees nothing
$\mathcal{H}$ is smooth

$\Rightarrow$ Like a Riadler horizon / eternal BH

$$\Rightarrow \quad \rho_{out} \approx e^{-\beta H}, \quad \beta = 8\pi M$$

= Hawking radiation!

This argument applies only to the <u>outgoing</u> modes



Conclude

* Physical state of QFT following BH collapse is $\approx |Unruh\rangle$

  This state is singular on the would-be past horizon, but that's OK b/c it does not exist!

* physical Schwarzschild BH's radiate @ $T = 1/8\pi M$

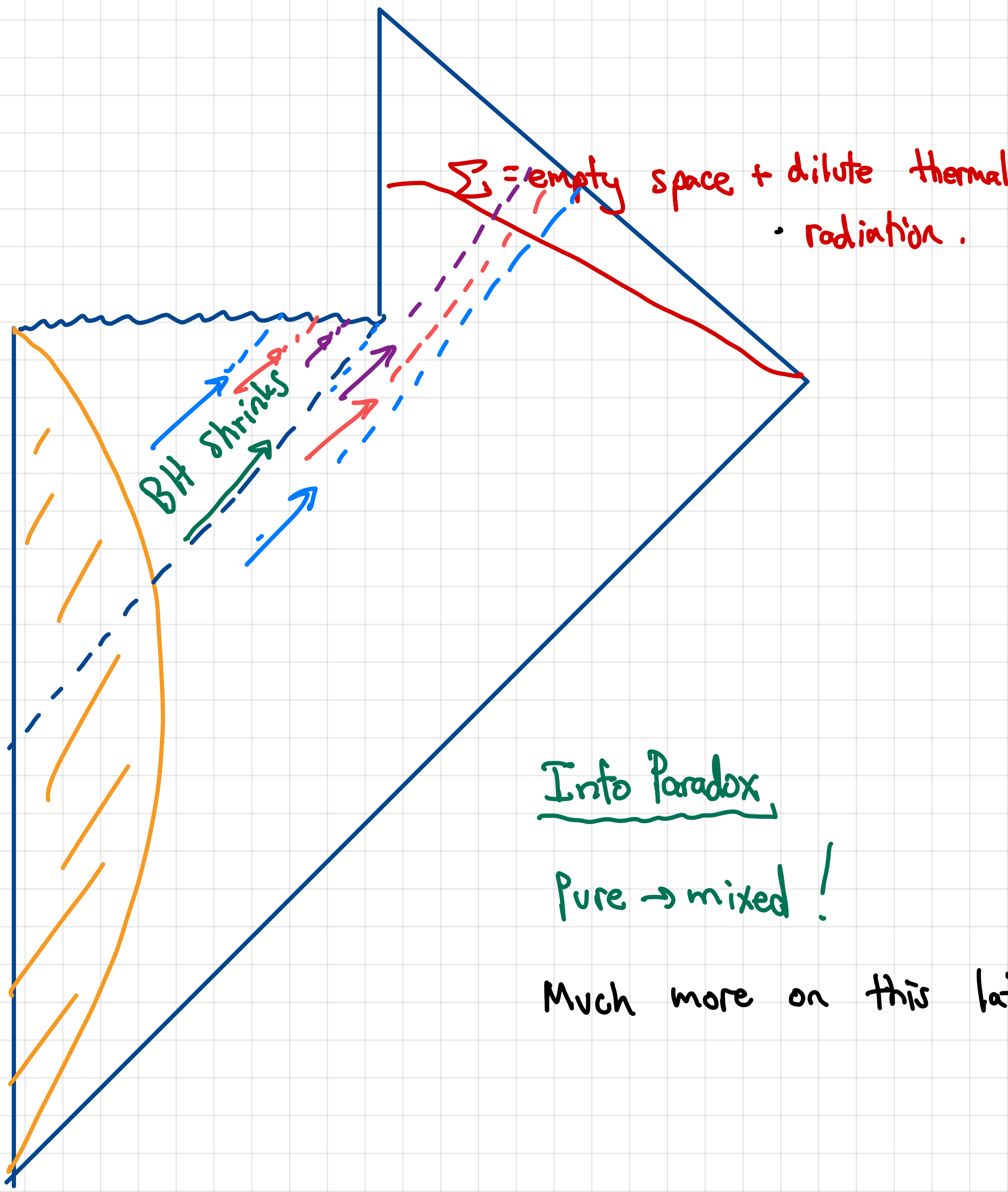
Emission rate

$$\Gamma_k = \underbrace{\frac{1}{e^{\beta\omega_k}-1}}_{\text{blackbody}} \; \underbrace{\sigma_{abs}(k)}_{\text{"greybody factor" to escape from near horizon to far away.}}$$

* Can also derive this using free field modes $a, a^\dagger$ etc. This is how Hawking did it - same answer. But not entirely justified as it requires following Planck-scale modes through the collapsing star (these are then redshifted to radiation @ $T_{Hawking}$)

The black hole loses energy to the radiation, so it shrinks very slowly.

Semiclassical Penrose Diagram



Info Paradox

Pure → mixed !

Much more on this later.